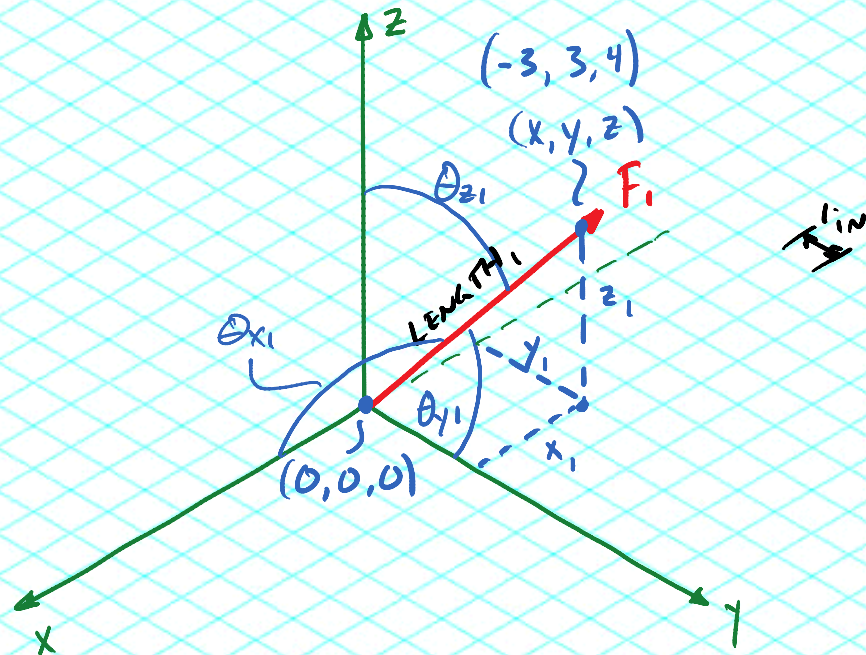


3D CONCURRENT FORCES

Monday, June 04, 2012
2:34 PM



$$L_1 = \text{LENGTH}_1 = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

$$= \sqrt{(-3\text{in})^2 + (3\text{in})^2 + (4\text{in})^2}$$

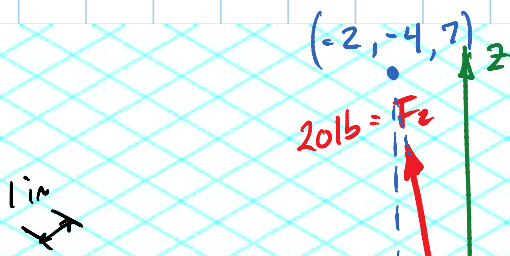
$$= 5.81\text{in}$$

$$\cos \theta_{y_1} = \frac{y_1}{L_1} \quad \therefore \theta_{y_1} = \cos^{-1}\left(\frac{y_1}{L_1}\right) = 58.9^\circ$$

$$\cos \theta_{x_1} = \frac{x_1}{L_1} \quad \therefore \theta_{x_1} = \cos^{-1}\left(\frac{x_1}{L_1}\right)$$

$$\cos \theta_{z_1} = \frac{z_1}{L_1} \quad \therefore \theta_{z_1} = \cos^{-1}\left(\frac{z_1}{L_1}\right)$$

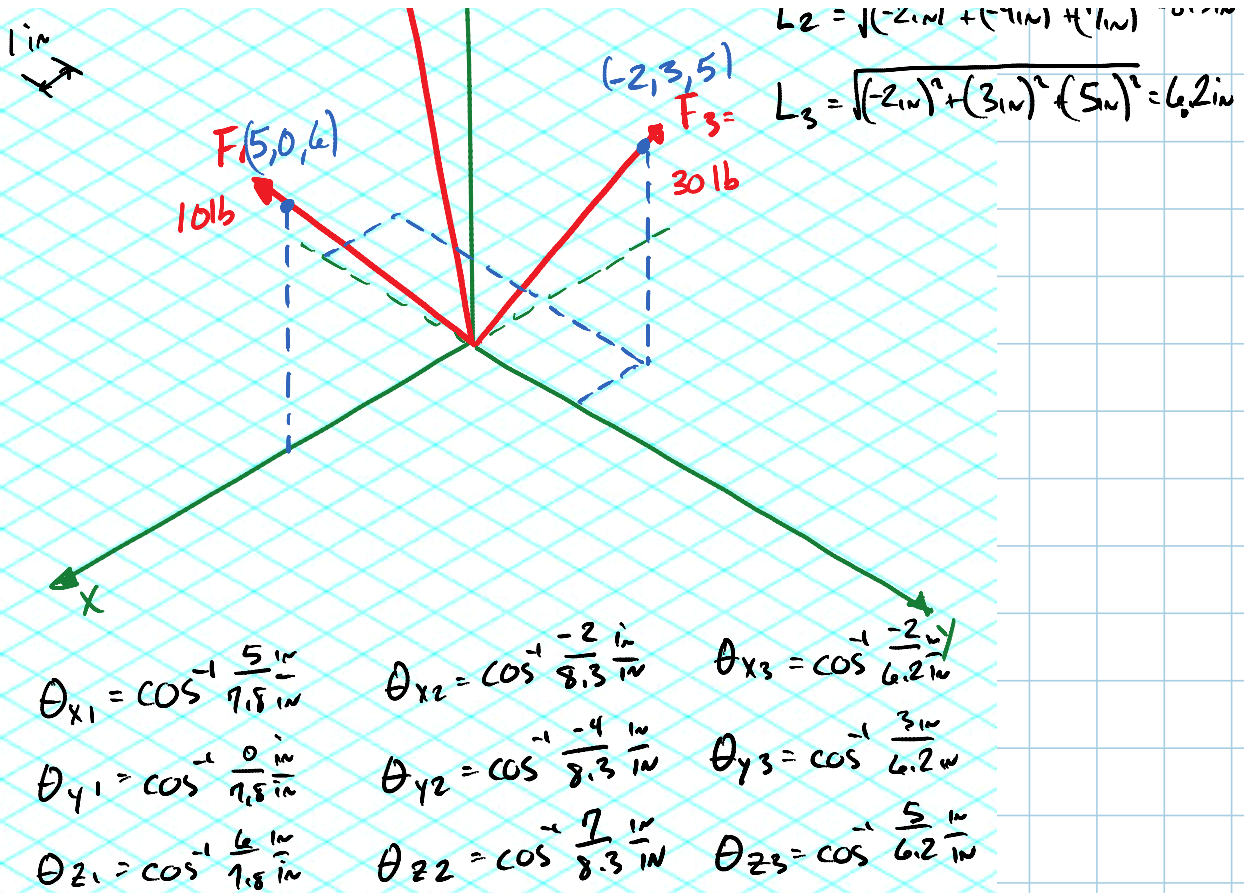
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$$L_1 = \sqrt{(5\text{in})^2 + 0 + (6\text{in})^2} = 7.81\text{in}$$

$$L_2 = \sqrt{(-2\text{in})^2 + (-4\text{in})^2 + (7\text{in})^2} = 8.31\text{in}$$

$$L_3 = \sqrt{(-2\text{in})^2 + (2\text{in})^2 + (5\text{in})^2} = 6.21\text{in}$$



$$\sum F_x = F_{1x} + F_{2x} + F_{3x} = R_x$$

$$= 10 \text{ lb} \cdot \cos \theta_{x1} + 20 \text{ lb} \cdot \cos \theta_{x2} + 30 \text{ lb} \cdot \cos \theta_{x3}$$

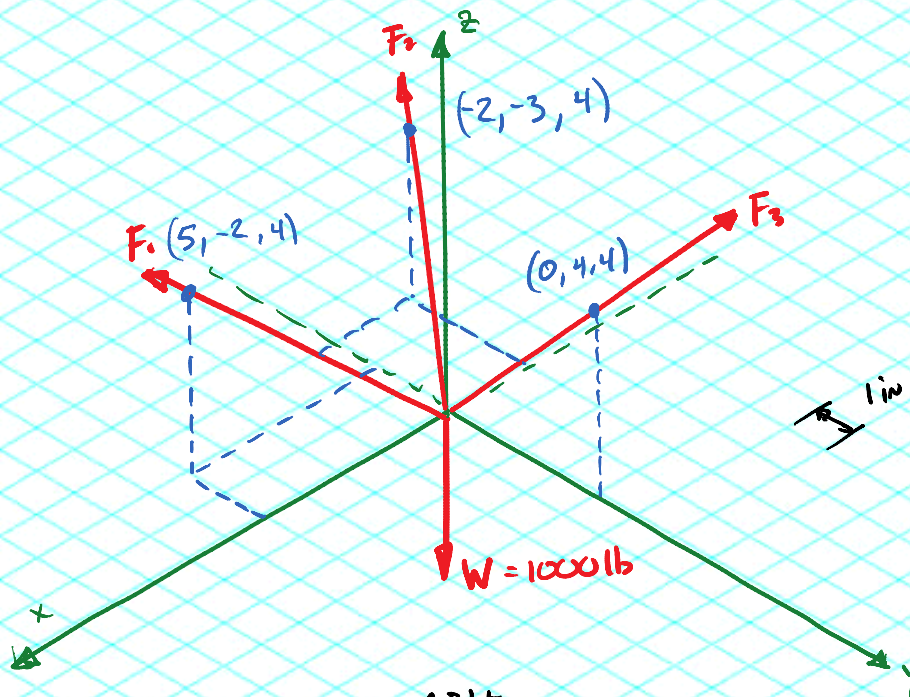
$$= 10 \text{ lb} \cdot \cos\left(\cos^{-1} \frac{5}{7.8}\right) + 20 \text{ lb} \cdot \cos\left(\cos^{-1} \frac{-2}{8.3}\right) + 30 \text{ lb} \cdot \cos\left(\cos^{-1} \frac{-2}{6.2}\right)$$

$$= 10 \text{ lb} \cdot \frac{5}{7.8} + 20 \text{ lb} \cdot \frac{-2}{8.3} + 30 \text{ lb} \cdot \frac{-2}{6.2}$$

$$= 6.4 \text{ lb} + (-4.8 \text{ lb}) + (-9.7 \text{ lb})$$

$$R_x = -8.1 \text{ lb}$$

3D EQUILIBRIUM



FIND! THE FORCE IN EACH CABLE.

$$L_1 = \sqrt{(5\text{in})^2 + (-2\text{in})^2 + (4\text{in})^2} = 6.7\text{ in}$$

$$L_2 = \sqrt{(-2\text{in})^2 + (-3\text{in})^2 + (4\text{in})^2} = 5.4\text{ in}$$

$$L_3 = \sqrt{0\text{in}^2 + (4\text{in})^2 + (4\text{in})^2} = 5.7\text{ in}$$

$$\theta_{x1} = \cos^{-1} \frac{5}{6.7}$$

$$\theta_{y1} = \cos^{-1} \frac{-2}{6.7}$$

$$\theta_{z1} = \cos^{-1} \frac{4}{6.7}$$

$$\theta_{x2} = \cos^{-1} \frac{-2}{5.4}$$

$$\theta_{y2} = \cos^{-1} \frac{-3}{5.4}$$

$$\theta_{z2} = \cos^{-1} \frac{4}{5.4}$$

$$\theta_{x3} = \cos^{-1} \frac{0}{5.7}$$

$$\theta_{y3} = \cos^{-1} \frac{4}{5.7}$$

$$\theta_{z3} = \cos^{-1} \frac{4}{5.7}$$

$$\begin{aligned} \Sigma F_x = R_x = 0 &= F_1 \cdot \cos \theta_{x1} + F_2 \cdot \cos \theta_{x2} + F_3 \cdot \cos \theta_{x3} \\ &= \frac{5}{6.7} \cdot F_1 + \left(\frac{-2}{5.4} \right) \cdot F_2 + \frac{0}{5.7} \cdot F_3 \end{aligned}$$

$$\begin{aligned} \Sigma F_y = R_y = 0 &= F_1 \cdot \cos \theta_{y1} + F_2 \cdot \cos \theta_{y2} + F_3 \cdot \cos \theta_{y3} \\ &= \frac{-2}{6.7} \cdot F_1 + \left(\frac{-3}{5.4} \right) \cdot F_2 + \frac{4}{5.7} \cdot F_3 \end{aligned}$$

Σ F_x = R_x = 0 = F₁ · cos θ_{x1} + F₂ · cos θ_{x2} + F₃ · cos θ_{x3}

$$\begin{aligned} \sum F_z = R_z = 0 &= F_1 \cdot \cos(\theta_{z1}) + F_2 \cdot \cos \theta_{z2} + F_3 \cdot \cos \theta_{z3} - 1000 \text{ lb} \\ &= \frac{4}{6.7} \cdot F_1 + \frac{4}{5.4} \cdot F_2 + \frac{4}{5.7} \cdot F_3 \end{aligned}$$

$$a = \text{COEFFICIENTS} = \begin{bmatrix} 5/6.7 & -2/5.4 & 0 \\ -2/6.7 & -3/5.4 & 4/5.7 \\ 4/6.7 & 4/5.4 & 4/5.7 \end{bmatrix} \begin{matrix} \leftarrow \sum F_x \\ \leftarrow \sum F_y \\ \leftarrow \sum F_z \end{matrix}$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ F_1 & F_2 & F_3 \end{matrix}$

$$b = \text{residuals} = \begin{bmatrix} 0 \\ 0 \\ 1000 \end{bmatrix} \begin{matrix} \leftarrow \sum F_x \\ \leftarrow \sum F_y \\ \leftarrow \sum F_z \end{matrix}$$

$\begin{matrix} \uparrow \\ 1000 \text{ lb} \end{matrix}$

$$\text{to solve: } a^{-1} \cdot b = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 285 \\ 575 \\ 576 \end{bmatrix} \text{ lb}$$